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WILSON POLYNOMIALS AND SOME CONTINUED FRACTIONS OF RAMANUJAN

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In honor of W.J. Thron on his 70th birthday.

ABSTRACT. We obtain the general solution to the recurrence relation for Wilson polynomials for the special cases $a+b+c+d=1,2,\ldots$. We derive a subdominant solution and, thus, from Pincherle's theorem, an explicit expression for the associated continued fraction and weight function. In the cases $a+b+c+d=1,2$ this yields the convergence properties of some continued fractions of Ramanujan. We also indicate how these results may be generalized to the q-Askey-Wilson case.

1. Introduction. Wilson polynomials form the most general class of orthogonal hypergeometric polynomials in the Askey-Wilson chart [**1**]. Here we examine properties associated with these polynomials by obtaining a subdominant solution to their recurrence relation

$$X_{n+1}-(z-a_n)X_n+b_n^2X_{n-1}=0 \tag{1}$$

and applying Pincherle's theorem to the corresponding continued fraction

$$CF(z)=z-a_0+\mathop{\mathbf{K}}_{n=1}^{\infty}\left(\frac{-b_n^2}{z-a_n}\right). \tag{2}$$

Definition 1. $X_n^{(s)}(z)$ is a subdominant solution of (1) at $z\in\mathbf{C}$ iff there exist linearly independent solutions $X_n^{(s)}(z)$, $X_n^{(d)}(z)$ with the property

$$\lim_{n\to\infty}X_n^{(s)}(z)/X_n^{(d)}(z)=0. \tag{3}$$

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