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PADÉ-TYPE APPROXIMANTS FOR FUNCTIONS OF MARKOV-STIELTJES TYPE

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ABSTRACT. The denominator of Padé approximants of functions of Markov-Stieltjes type are closely connected to orthogonal polynomials which leads to control of the location of the poles and to convergence of the approximants. We investigate to which extent the convergence also holds for Padé-type approximants where the location of the poles is changed.

0. Introduction. Let f be a function of the following type (Markov-Stieltjes type)

$$f(z) = \int_{-1}^{1} \frac{d\alpha(t)}{1+zt} \tag{0.1}$$

where $z \in \mathbf{C}$, the complex plane, and α is a finite positive measure whose support is an infinite subset of $[-1, 1]$. Let P_{n-1}/Q_n be the $(n-1, n)$ Padé approximant of f, i.e., P_{n-1} and Q_n, $Q_n \not\equiv 0$, are polynomials of degree at most $n-1$ and n, respectively, satisfying the following interpolation condition at zero

$$(fQ_n - P_{n-1})(z) = O(z^{2n}) \quad \text{as } z \to 0. \tag{0.2}$$

Then $q_n(z) := z^n Q_n(-1/z)$ is the n-th degree orthogonal polynomial for α (see Section 2.1). This means that the zeros of $Q_n(z)$ are simple and located on $]-\infty, -1[\,\cup\,]1, \infty[$, and from this it can be proved that $P_{n-1}/Q_n \to f$ locally uniformly in $\mathbf{C}\backslash(]-\infty, -1] \cup [1, \infty[)$ (Markov's theorem [**8**]) with geometric degree of convergence (Gragg; see, for instance, [**6**]). Furthermore, if α is absolutely continuous and $\alpha'(x) > 0$ almost everywhere in $[-1, 1]$, the zeros of $q_n(z)$ are distributed asymptotically according to the *arcsine* distribution, i.e., according to the equilibrium distribution of $[-1, 1]$ for the logarithmic potential

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