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ON THE REMAINDER TERM FOR ANALYTIC FUNCTIONS OF GAUSS-LOBATTO AND GAUSS-RADAU QUADRATURES

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Dedicated to Wolfgang Thron on his 70th birthday

ABSTRACT. We study the kernels in the contour integral representation of the remainder term of Gauss-Lobatto and Gauss-Radau quadratures, in particular the location of their maxima on circular and elliptic contours. Quadrature rules with Chebyshev weight functions of all four kinds receive special attention, but more general weights are also considered.

1. Introduction. Let Γ be a simple closed curve in the complex plane surrounding the interval $[-1,1]$ and $\mathcal{D}$ be its interior. Let f be analytic in $\mathcal{D}$ and continuous on $\overline{\mathcal{D}}$. We consider an interpolatory quadrature rule

$$\int_{-1}^{1} f(t)w(t)\,dt = \sum_{\nu=1}^{N} \lambda_\nu f(\tau_\nu) + R_N(f) \tag{1.1}$$

with

$$-1 \le \tau_N < \tau_{N-1} < \cdots < \tau_1 \le 1 \tag{1.2}$$

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$$\omega_N(z) = \omega_N(z;w) = \prod_{\nu=1}^{N}(z-\tau_\nu), \quad z \in \mathbf{C}, \tag{1.3}$$

denote its node polynomial (which in general depends on w), and define

$$\rho_N(z;w) = \int_{-1}^{1} \frac{\omega_N(t;w)}{z-t} w(t)\,dt, \quad z \in \mathbf{C}\backslash[-1,1], \tag{1.4}$$

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