ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 21, Number 1, Winter 1991

MODEL REDUCTION AND STABILITY OF TWO-DIMENSIONAL RECURSIVE SYSTEMS

ANNIE CUYT[1,2] WILLIAM B. JONES[3] AND BRIGITTE VERDONK[1]

ABSTRACT. Many approaches can be found in the literature for the realization of two-dimensional recursive digital filters using both ordinary [**12, 17, 18**] and branched continued fraction expansions [**3, 4**]. In this paper we introduce a new type of branched continued fraction expansion which has specific advantages when considering the model reduction problem. The form of the new BCF is such that, when the expansion is constructed for the transfer function of a stable system, convergents of the BCF expansion automatically satisfy one part of the Huang stability theorem [**6, 9**]. In Section 1 we shall first briefly review the one-dimensional case, and in Section 2 we shall give the algorithm for the new BCF expansion and indicate how the simplification of the stability test for the "reduced" systems follows in a natural way. We conclude Section 2 with an example.

1. One-dimensional recursive systems. Consider a one-dimensional linear shift-invariant (LSI) recursive system T satisfying a finite difference-equation of the form

$$y_n = \sum_{k=0}^{N} a_k x_{n-k} - \sum_{k=1}^{M} b_k y_{n-k}. \tag{1}$$

Then it is well known [**15**] that the transfer function $H(z)$ of the system is a rational function given by

$$H(z) = \frac{\sum_{k=0}^{N} a_k z^{-k}}{\sum_{k=0}^{M} b_k z^{-k}}, \qquad b_0 = 1. \tag{2a}$$

[1] This research was done while the first and third authors were visiting the University of Colorado, Boulder.

[2] Research Associate NFWO.

[3] Research supported in part by the U.S. National Science Foundation under Grant Number DMS-8700498.

Received by the editors on August 1, 1988, and in revised form on December 9, 1989.