

**THE ASYMPTOTIC BEHAVIOR OF SEQUENCES AND
NEW SERIES TRANSFORMATIONS BASED
ON THE CAUCHY PRODUCT**

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ABSTRACT. We first give a review of results concerning the asymptotic behavior of the ratio of the errors and the ratio of the differences for a convergent sequence. Then a review and new results on the asymptotic comparison of ratios of errors and ratios of differences for two converging sequences are given. These results are used for showing how to accelerate the convergence. In particular, new series transformations based on the Cauchy product by an arbitrary given series are discussed and their properties are studied.

Introduction. The construction and the study of convergence acceleration methods for sequences and series needs the knowledge of results on the asymptotic behavior of the ratio of the errors and the ratio of the differences between two consecutive terms of a convergent sequence. It also needs some results on the asymptotic behavior of ratios of the errors and ratios of the differences for two sequences. In the first two sections we shall review such existing results and give some new ones. In the last section we shall use them to show how to accelerate the convergence under certain assumptions. Recently, some old results on the ratio of a term of a series obtained by Cauchy product from a previous one divided by the corresponding term of the initial series were rediscovered and extended by using techniques of nonstandard analysis [2]. These results are used to build new series transformations leading to the concept of Cauchy-type approximation. This type of approximation is related to Padé-type approximation in a particular case. Some acceleration properties of this transformation are given and an extension is studied.

Asymptotic behavior of a sequence. We shall give a review of the existing results. (u_n) is a sequence converging to u . The most general result is

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