BOUNDS ON THE ORDER OF GENERATION
OF SO(n, R) BY ONE-PARAMETER SUBGROUPS

F. SILVA LEITE

ABSTRACT. A Lie group $G$ is said to be uniformly finitely generated by one-parameter subgroups $\exp(tX_i)$, $i = 1, \ldots, n$, if there exists a positive integer $k$ such that every element of $G$ may be expressed as a product of at most $k$ elements chosen alternatively from these one-parameter subgroups.

In this paper we construct sets of left invariant vector fields on $SO(n)$, in particular, pairs $\{A, B\}$, whose one-parameter subgroups uniformly finitely generate $SO(n)$ and find an upper bound on the order of generation of $SO(n, R)$ by these subgroups. We give special attention to the case $n = 3$.

0. Introduction. If the Lie algebra of a connected Lie group $G$ is generated by the elements $X_1, \ldots, X_n$, then every element of $G$ may be expressed as a finite product of elements of the form $\exp(tX_i)$, where $t$ is real and $i = 1, \ldots, n$ (Jurdjevic and Sussmann [6]). However, the number of elements required for $g \in G$ may not be uniformly bounded as $g$ ranges through $G$. If, in addition, $G$ is compact and $\exp(tX_i)$, $i = 1, \ldots, n$ are also compact, then it follows from Theorem 1.1 that there exists a positive integer $k$ such that every element of $G$ may be expressed as a product of at most $k$ elements from $\exp(tX_i)$, $i = 1, \ldots, n$. That is, $G$ is uniformly finitely generated by these one-parameter subgroups with order of generation $k$.

For two and three-dimensional Lie groups, the problem has been completely solved by Koch and Lowenthal. In [1], Crouch and the present author take the initial steps in the problem of uniform finite generation of $SO(n, R)$ (the real $n(n-1)/2$-dimensional special orthogonal group with Lie algebra $so(n)$) and concentrate on finding pairs of generators for $so(n)$, orthogonal with respect to the killing form $\langle \cdot, \cdot \rangle$ and whose one-parameter subgroups uniformly finitely generate $SO(n)$.

This paper is still devoted to the uniform generation problem of $SO(n)$. Section 1 is introductory. Sections 2 and 3 are concerned with