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BOUNDS ON THE ORDER OF GENERATION OF SO(n, R) BY ONE-PARAMETER SUBGROUPS

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ABSTRACT. A Lie group G is said to be uniformly finitely generated by one-parameter subgroups $\exp(tX_i)$, $i = 1, \dots, n$, if there exists a positive integer k such that every element of G may be expressed as a product of at most k elements chosen alternatively from these one-parameter subgroups.

In this paper we construct sets of left invariant vector fields on $SO(n)$, in particular, pairs $\{A, B\}$, whose one-parameter subgroups uniformly finitely generate $SO(n)$ and find an upper bound on the order of generation of $SO(n, \mathbf{R})$ by these subgroups. We give special attention to the case $n = 3$.

0. Introduction. If the Lie algebra of a connected Lie group G is generated by the elements $X_1, \dots, X_n$, then every element of G may be expressed as a finite product of elements of the form $\exp(tX_i)$, where t is real and $i = 1, \dots, n$ (Jurdjevic and Sussmann [**6**]). However, the number of elements required for $g \in G$ may not be uniformly bounded as g ranges through G. If, in addition, G is compact and $\exp(tX_i)$, $i = 1, \dots, n$ are also compact, then it follows from Theorem 1.1 that there exists a positive integer k such that every element of G may be expressed as a product of at most k elements from $\exp(tX_i)$, $i = 1, \dots, n$. That is, G is uniformly finitely generated by these one-parameter subgroups with order of generation k.

For two and three-dimensional Lie groups, the problem has been completely solved by Koch and Lowenthal. In [**1**], Crouch and the present author take the initial steps in the problem of uniform finite generation of $SO(n, \mathbf{R})$ (the real $n(n-1)/2$-dimensional special orthogonal group with Lie algebra $so(n)$) and concentrate on finding pairs of generators for $so(n)$, orthogonal with respect to the killing form $\langle \cdot, \cdot \rangle$ and whose one-parameter subgroups uniformly finitely generate $SO(n)$.

This paper is still devoted to the uniform generation problem of $SO(n)$. Section 1 is introductory. Sections 2 and 3 are concerned with

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