ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 21, Number 2, Spring 1991

SOLUTIONS OF A NONLINEAR BOUNDARY LAYER PROBLEM ARISING IN PHYSICAL OCEANOGRAPHY

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ABSTRACT. We investigate a mathematical model for large scale ocean circulation. Under reasonable assumptions the partial differential equations reduce to the third order ordinary differential equation $\phi''' + \lambda(\phi\phi'' - (\phi')^2) + 1 - \phi = 0$ with either "no-slip" initial conditions $\phi(0) = 0$, $\phi'(0) = 0$ or "stress-free" initial conditional $\phi(0) = 0$, $\phi''(0) = 0$. The appropriate boundary condition in each case is $\phi(\infty) = 1$. We prove that for each $\lambda \geq (27/4)^{1/3}$, the no slip problem and the stress free problem each has at least one solution.

I. Introduction. We investigate the existence of solutions of the equation

$$\phi''' + \lambda(\phi\phi'' - (\phi')^2) + 1 - \phi = 0 \tag{1}$$

which satisfy either of the initial conditions

$$\phi(0) = \phi'(0) = 0 \tag{2}$$

or

$$\phi(0) = \phi''(0) = 0, \tag{3}$$

and subject to the boundary condition

$$\phi(\infty) = 1. \tag{4}$$

Equation (1) arises in the theory of physical oceanography and was developed by Ierley and Ruehr [**2**]. They derived a two-dimensional, one layer model for large scale ocean circulation with particular emphasis on the gulf stream. They assume that the steady state vorticity equation holds and restrict x and y to a rectangular region. Taking into account the east-west flow of the wind, they observe that a boundary layer

Received by the editors on May 26, 1987.
This research was partially supported by NSF Grant No. DMS-8501531.