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A HYPERBOLIC STEFAN PROBLEM

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1. Introduction. In order to utilize the balance of energy to calculate temperatures, it is necessary to introduce constitutive assumptions relating to energy storage, e, and flux, $\vec{q}$, to the temperature u. Traditionally, e is chosen proportional to u and $\vec{q}$ proportional to ∇u. This leads to the classical parabolic heat equation, where a thermal disturbance is instantly felt throughout the body. Other admissible constitutive relations allow e and q to depend upon the temperature history [**9**],

$$\vec{q}(t) = -\int_0^\infty a(s)\nabla u(t-s)\,ds.$$

Selecting $a(s) = (k/\tau)e^{-(s/\tau)}$ in the above (cf. [**9**]) yields

$$\left(1+\tau\frac{d}{dt}\right)\vec{q} = -k\nabla u, \tag{1.1}$$

which leads to the hyperbolic telegraphers equation

$$\tau c u_{tt} + c u_t - k\Delta u = 0 \tag{1.2}$$

in place of the classical heat equation ($\tau = 0$). Such a model will certainly have an upper bound on the speed of thermal disturbances.

We shall formulate a well-posed free-boundary problem of the Stefan type [**7,13–15**] consistent with (1.1) and which contains the telegraphers equation (1.2).

2. Energy and phase change models. If a block of ice of unit volume is subject to a uniform heat source of intensity $F > 0$, classical models predict a temperature increase at the rate of F/c_1 until the temperature reaches $u = 0$. While the ice melts the temperature remains at zero and the fraction of water, ξ, increases from 0 to 1 at a

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