ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 21, Number 2, Spring 1991

SINGULAR LIMIT APPROACH TO STABILITY AND BIFURCATION FOR BISTABLE REACTION DIFFUSION SYSTEMS

YASUMASA NISHIURA

1. Introduction. Patterns with sharp transition layers appear in various fields such as patchiness and segregation in ecosystems [**3,10**], traveling waves in excitable media [**2,4,5,** and **22**], striking patterns in morphogenesis models [**13**], dendric patterns in solidification problem [**1**], and so on.

The most simple but substantial model system, to which most of the above ones fall, is given by the following reaction-diffusion equations in one-dimensional space:

$$\text{(P)} \qquad \begin{aligned} u_s &= d_1 u_{xx} + f(u,v) \\ v_s &= d_2 v_{xx} + \delta g(u,v) \end{aligned} \qquad \text{on } I,$$

where d_1 and d_2 are the diffusion rates of u and v, and δ is the ratio of the reaction rates. The interval I is either $(-l, l)$ or $\mathbf{R}$. The Neumann boundary conditions $u_x = 0 = v_x$ is added to (P), when $I = (-l, l)$. It is usually assumed in (P) that one of the following conditions holds:

(a) There is a significant difference in the diffusion rates of u and v.

(b) There is a significant difference in the reaction rates of u and v.

(c) There is a combination of (a) and (b).

Most of the symmetry breaking stationary patterns in the framework of Turing's diffusion driven instability fall into the first category. One of the well-known models is the Gierer and Meinhardt equation describing morphogenetic patterns [**13**]. Propagator-controller systems including a simple skeleton model for the B-Z reaction lie on the second category [**23, 5**]. It is essential for such systems that one of the components reacts much faster than the other. Formally speaking, the FitzHugh-Nagumo equations belong to the third category in which the second component v does not diffuse and reacts much slower than the first one [**8**]. However, the qualitative behavior of solutions of the FHN

Received by the editors on April 4, 1988, and in revised form on August 5, 1989.