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GLOBAL EXISTENCE FOR SEMILINEAR PARABOLIC SYSTEMS ON ONE-DIMENSIONAL BOUNDED DOMAINS

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ABSTRACT. We consider semilinear parabolic systems of partial differential equations of the form

$$u_t(x,t) = Du_{xx}(x,t) + Cu_x(x,t) + f(u(x,t)) \quad 0 < x < 1,\ t > 0$$

with bounded initial data and homogeneous Dirichlet boundary conditions, where D is an m by m diagonal positive definite matrix, C is an m by m diagonal matrix and $f : \mathbf{R}^m \to \mathbf{R}^m$ is locally Lipschitz. We prove that if the vector field f satisfies a generalized Lyapunov type condition, then solutions of (1) exist for all $t > 0$. Our result begins an extension of recent results in Morgan [**7**].

1. Introduction and notation. Until recent years, most of the work on semilinear parabolic systems of partial differential equations has fallen into one of two groups; one either assumes that sufficient a priori bounds can be obtained for solutions of the system or assumes a bounded invariant region exists for the system. Of these two approaches, generally only the second considers the vector field involved as anything more than an algebraic expression. Consequently, since invariant regions do not exist for many systems, the geometry of the vector field involved is often ignored. Recently, however, Alikakos [**1**], Groger [**3**], Hollis, Martin, and Pierre [**4**], Masuda [**6**], and others have begun to exploit this geometry via Lyapunov type structures. Some of their results are extended in [**7**]. In this work we extend the results in [**7**] to include systems containing linear convection terms. For simplicity, we restrict our attention to the one-dimensional setting. A complete extension of these results in arbitrary space dimensions, including an interesting treatment of a class of systems with nonlinear diffusion coefficients, will be given in a forthcoming paper of Fitzgibbon, Morgan and Waggoner.