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STUDYING SINGULAR SOLUTIONS OF A SEMILINEAR HEAT EQUATION BY A DILATION RESCALING NUMERICAL METHOD

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ABSTRACT. A method of dynamic rescaling of variables is used to investigate numerically the nature of the point singularities of the cubic and quadratic nonlinear heat equations in one and two dimensions, with evenness and radial symmetry, respectively. This has allowed solutions to be computed until the amplitude of the spike has grown by a factor of 10^9 or more.

This high numerical resolution is used first to corroborate the occurrence of singularities of the predicted form for several choices of initial data and then to test a conjecture of Galaktionov and Posashkov [**4**] concerning the spatial scale and shape of the solution near the singularity.

1. Background. The equation

(1.1)
$$\phi_t=\Delta\phi+|\phi|^{p-1}\phi, \quad \phi:\mathbf{R}^+\times\Omega\to\mathbf{R}, \quad \Omega\subseteq\mathbf{R}^d, \quad p>1 \quad \phi(0,x)=\phi_0(x)$$

is known to develop point singularities in finite time (Fujita [**2,3**]). It also has the trivial singular solutions

(1.2)
$$\phi(t,x)=\kappa(t^*-t)^{-\beta}, \qquad \beta=1/(p-1), \qquad \kappa=\beta^\beta.$$

It is conjectured that the growth rate of all singular solutions is the same as for the ODE solutions (1.2). The "ODE growth rate" can be shown to be a lower bound using local in time existence theory. It has been shown to be an upper bound also under numerous combinations of extra hypotheses by Weissler [**10**], Friedman and McLeod [**1**], and Giga and Kohn [**5,6**].

The latter result is related to a rescaling to $u(\tau,\xi)$

(1.3a)
$$u=\lambda^{-2\beta}(t)u, \qquad \xi=x/\lambda(t), \qquad d\tau/dt=\lambda^{-2}(t)$$

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