ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 21, Number 2, Spring 1991

THE ONE-DIMENSIONAL DISPLACEMENT IN AN ISOTHERMAL VISCOUS COMPRESSIBLE FLUID WITH A NONMONOTONE EQUATION OF STATE

K. KUTTLER AND D. HICKS

The one-dimensional conservation laws of volume and momentum [**10**] may be written as

$$\frac{\partial V}{\partial t} = \frac{\partial v}{\partial x}, \qquad \frac{\partial v}{\partial t} = \frac{-\partial S}{\partial x} \tag{1.1}$$

where V is a specific volume, v the velocity and S the stress. In this paper it will be assumed that $S = P + q$ where P is the pressure and q is the part of the stress due to viscosity. It is also assumed that the internal energy is constant. Thus, it seems reasonable that

$$P = P(V), \qquad q = -\alpha(V)\frac{\partial v}{\partial x}, \qquad \alpha(V) > 0, \tag{1.2}$$

where $\alpha(V)$ is a coefficient of viscosity. Define the displacement, U, by

$$U(t,x) = \int_0^t v(s,x)\,ds. \tag{1.3}$$

Thus $U_t = v$, $U_{tt} = v_t$, $U_{tx} = v_x$, and

$$U_x(t,x) = \int_0^t v_x(s,x)\,ds = \int_0^t V_t(s,x)\,ds = V(t,x) - V_0(x). \tag{1.4}$$

Substituting this into the second of the equations of (1.1) and allowing for a body force yields the equation for U,

$$U_{tt}(t,x) + (P(V(t,x)))_x - (\alpha(V(t,x))U_{tx}(t,x))_x = g(t,x), \tag{1.5.1}$$

where $V(t,x) = V_0(x) + U_x(t,x)$ and g comes from the body force. Letting $U_1(x) = U_t(0,x)$ (1.3) yields the initial conditions

$$U(0,x) = 0, \tag{1.5.2}$$

$$U_t(0,x) = U_1(x). \tag{1.5.3}$$

Received by the editors on June 1, 1987, and in revised form on March 2, 1988.