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A MATHEMATICAL PROBLEM IN GEOCHEMISTRY: THE REACTION-INFILTRATION INSTABILITY

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1. Introduction.

When reactive waters flow through a porous medium they can dissolve the minerals and cause changes in porosity. This, through Darcy's law, can alter the flow, giving rise to a feedback mechanism which can cause instabilities in the shape of the porosity level surfaces. This mechanism most certainly is important in many geochemical situations (e.g., the diagenesis and evolution of mineral, oil and gas reservoirs, the dynamics of nuclear and chemical waste repositories, in situ coal gasification, enhanced oil recovery, etc.). Our own interest in the subject arose from trying to understand the occurrence of so-called roll-front redox mineral deposits [**1,3,4**]. No doubt the coupling of this reaction-infiltration instability to the more widely studied multi-phase flow instabilities should lead to a very rich area for further research.

Typically, the essential geochemical processes of relevance to each of the above situations can be modelled mathematically as a system of coupled, highly nonlinear reaction-transport equations [**3,4**]. In general, however, even the simplified versions of these equations arising from overly simplified physical models are too complicated to be studied abstractly or analytically [**5**]. Our approach here (Section 2) is to restrict attention to a physically important class of problems for which the effective reaction zone (where the serious complications appear) is much smaller and less interesting than the scale of the phenomenon being studied. The resulting set of reaction-transport equations can then be studied using matched asymptotics [**5**] to obtain a more amenable moving free boundary problem for the reaction interface [**4,5**] (Section 3). This will allow us to give [**1,5,6**] (Section 4) a mathematical treatment of the evolution of the shape of the reaction interface in terms

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