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NODAL PROPERTIES OF SOLUTIONS OF PARABOLIC EQUATIONS

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1. Introduction. In this note we review the known facts about the zero set of a solution of a scalar parabolic equation

$$(1)\quad u_t = a(x,t)u_{xx} + b(x,t)u_x + c(x,t)u, \qquad x_0 < x < x_1, 0 < t < T.$$

In particular, we discuss some applications to spectral theory, the dynamics of nonlinear diffusion equations, and the geometric heat equation for plane curves.

2. The zero number. Let u be a classical solution of (1) and assume u is continuous on the rectangle $[x_0, x_1] \times [0, T]$. Moreover, assume that

$$u(x_i, t) \neq 0 \quad \text{for } i = 0, 1 \quad \text{and} \quad 0 \leq t \leq T.$$

Then, for each $t \in [0, T]$ we define the set $Z(t) = \{x \in [x_0, x_1] \mid u(t, x) = 0\}$, and we let $z(t)$ denote the number of elements of $Z(t)$. The set $Z(t)$ is a compact subset of the open interval (x_0, x_1).

Finally, we always assume the following about the coefficients a, b and c:

$$(2)\quad \begin{array}{l} a, a_x, a_{xx}, a_t, b_x, b_t \text{ and } c \text{ are continuous on } [x_0, x_1] \times [0, T]. \\ \text{Moreover, } a(x,t) \text{ is strictly positive.} \end{array}$$

In this situation we have the following:

Theorem A. *For any $0 < t \leq T$, $z(t)$ is finite. If, for some $0 < t_0 < T$, the function $u(t_0)$ has a double zero, then for all*

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