ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 21, Number 3, Summer 1991

A SIMPLE CONTINUED FRACTION TEST FOR THE IRRATIONALITY OF FUNCTIONS

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ABSTRACT. A theorem on irrationality of positive continued fractions with one variable is proved by introducing the behavior of tails.

Introduction. The subject for study in this paper is the continued fraction of the type

$$\operatorname*{K}_{n=1}^{\infty} \frac{a_n(x)}{b_n(x)} \tag{1}$$

where the elements $a_n(x)$ and $b_n(x)$ are nonzero polynomials of the variable $x > 0$ with nonnegative coefficients. The continued fractions

$$T_k(x) = \operatorname*{K}_{n=k}^{\infty} \frac{a_n(x)}{b_n(x)} \tag{2}$$

for $k \geq 1$ are called the tails of (1). As background to the problem of irrationality of functions by the aid of continued fraction theory we refer to [**1**; Corollary 5.3, p. 156 and p. 380] where further references are given. In the most known irrationality tests for functions where continued fractions are used, there is no involvement of convergence and the behavior of tails considered as functions. For an example, see the C-fraction test in [**1**, p. 156]. In the present paper we give an example of a theorem on irrationality where both convergence and the magnitude of the tails play a crucial role. The basic idea is quite general and flexible, but we apply it only on the continued fraction of type (1) for the sake of simplicity. An example of a continued fraction of type (1) is

$$x = \frac{x+1}{1} \underset{+}{} \frac{2}{x} \underset{+}{} \frac{x+1}{1} \underset{+}{} \cdots \tag{3}$$

Received by the editors on March, 2, 1987.