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STEADY-STATE TURBULENT FLOW WITH REACTION

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ABSTRACT. Existence and uniqueness of nonnegative solutions of the two-point boundary value problem $\psi(\phi(u)')' = f(x,u,u'), u(-1) = a, u(1) = b$ are established for appropriate functions ϕ, ψ, and f. Included in this formulation are the one-dimensional steady-state equations for turbulent or diffusive flow in a porous catalytic pellet, irreversible reaction with change of volume, etc. Also examined is the possibility that the concentration u might vanish on some nontrivial subset of $[-1,1]$, the dead core.

Introduction. A mathematical description for one-dimensional turbulent flow of a polytropic gas in a porous medium has been given by Leibenson [**8**]; cf. Esteban and Vazquez [**5**]. If the gas is being consumed in the medium through undergoing an irreversible reaction, then the steady-state concentration u is described by the nonlinear differential equation

$$\frac{d}{dx}\Big(\frac{du^q}{dx}\Big|\frac{du^q}{dx}\Big|^{p-1}\Big) = \lambda f(u).$$

Here the constants p and q satisfy $1/2 \leq p \leq 1$ and $q \geq 2$ in the physical problem, the Thiele modulus λ is a positive constant (essentially reaction rate divided by diffusion rate), and $f > 0$ specifies the nature of the reaction. If we assume that the porous catalyst occupies the region $-1 \leq x \leq 1$, then a reasonable problem arises on specifying Dirichlet boundary conditions

$$u(-1) = a \geq 0, \qquad u(1) = b \geq 0.$$

For physical reasons we are interested only in nonnegative u.

This problem can be recast in the more general form

$$\frac{d}{dx}\psi\Big(\frac{d}{dx}\phi(u)\Big) = \lambda f(u), \qquad u(-1) = 1, u(1) = b,$$

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