ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 21, Number 3, Summer 1991

ORTHOMODULAR LATTICES AND QUADRATIC SPACES: A SURVEY

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Dedicated to the Memory of Charles H. Randall, mathematician and friend

Introduction. The theory of quadratic and Hermitian forms has a long and fruitful history in mathematics. From the time of Fermat to the time of Minkowski, quadratic forms belonged to number theory and an impressive arithmetic theory of forms was developed. In 1937, Witt broke new ground bringing the theory of forms into a more modern algebraic setting emphasizing classification and general structure. In 1967, Pfister demonstrated the power of this approach when he published his celebrated structure theorems. To this day, mathematicians are uncovering the beauties of this algebraic theory (see the recent book of Scharlau [**B20**]).

There is an area of mathematics, not in the mainstream described in the paragraph above, where quadratic and sesquilinear forms have also made a significant, though perhaps unexpected, contribution in recent years. This is in the theory of orthocomplemented lattices, especially those known as "orthomodular." The story goes back (at least) to the seminal 1936 paper of Birkhoff and von Neumann [**10**] which has led to much research in what is commonly called the "logic of quantum mechanics" (Math Reviews classifies this area under 81B10). This historic paper argued against the classical Boolean algebra structure of logic identifying the distributive law as being untenable in the logic of empirically verifiable propositions concerning a quantum mechanical system. These authors argued for the structure of a projective geometry (essentially an orthocomplemented modular lattice). Subsequent researchers have generalized further to an orthomodular partially ordered set saying even a lattice structure is too much. In his monograph, Mackey [**B15**] presented some physically plausible axioms concerning states and observables and derived a logic which was a σ-orthocomplete orthomodular poset. He then postulated that this logic was orthoisomorphic to the lattice of closed subspaces of a separable

Received by the editors on October 2, 1987.