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CONSTRICTED SYSTEMS

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0. Introduction. Recent interest in the investigation of chaotic behavior of dynamic systems has led to a broad renewed interest in L_1 Markov operators. The recent monograph of Lasota and Mackey [**11**] gives a very readable introduction to the methods and applications of this approach to the randomness of deterministic processes. Lasota, Li, and Yorke [**10**] and Lasota and Yorke [**12**] have studied the asymptotic periodicity of an L_1 Markov operator which has a constricting set which attracts densities. For strongly constricted systems we obtain asymptotic finite dimensionality for a contraction on any B-space. Similar results are obtained for weakly compact constrictors on B-spaces for which the geometry can be exploited. Bartoszek [**1**] has extended the strong constrictor results of [**10**] to positive operators on arbitrary Banach lattices. The approach here has some advantages in that positivity is not required and the B-space can be arbitrary.

Using a clever and elementary argument Komornik [**7**] has shown that weak implies strong for a constricted L_1 Markov operator. While the deLeeuw–Glicksberg machinery we bring to bear does give some results in an effortless fashion, it does not give this result. We give an example of a positive isometry of $C(X)$ which is weakly but not strongly constricted to show that a full strength Komornik Theorem is not possible in $C(X)$.

1. Let T be a linear contraction on a B-space $\mathbf{X}$. Suppose F is a compact subset of $\mathbf{X}$ with the property that for each x in the unit ball, $B(\mathbf{X})$,

$$\operatorname{dist}(T^n x, F) \to 0.$$

We will call F a (norm or strong) *constrictor* for T. Note that we have not assumed that F is convex or invariant under the action of T. The first defect is easily fixed, for if F is a constrictor for T so is the norm closed convex hull of F. The second defect is also easily fixed—indeed

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