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ON A QUASILINEAR DEGENERATE HYPERBOLIC SYSTEM OF CONSERVATION LAWS DESCRIBING NONLINEAR ADVECTION PHENOMENA

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ABSTRACT. We consider a two-dimensional system of conservation laws which is hyperbolic but degenerate, for either characteristic field is genuinely nonlinear in one half of the phase-plane and linearly degenerate in the other half. We prove the existence and uniqueness of the solution of the Riemann problem and the existence of a (BV) solution of the initial-value problem. This system arises in modelling certain nonlinear advection processes and, as shown by the support properties we establish in case of special initial data, may describe pattern differentiation.

0. Introduction. The object of the present paper is to study the system of conservation laws

$$\begin{bmatrix} u_t + (u(1-v))_x = 0 \\ v_t + (v(1+u))_x = 0 \end{bmatrix} \text{ in } \mathbf{R} \times \mathbf{R}^+. \tag{0.1}$$

This system arises if we consider u, v as the space derivatives of non-negative quantities representing the densities of two populations, the *fugitives* (denoted by $U(x,t)$) and the *pursuers* (denoted by $V(t,x)$). According to a model originally proposed by Murray and Cohen [**12**], we may characterize a pursuing-escape interaction with predation along a straight line course by the equations

$$\begin{bmatrix} U_t + (U(1-V_x))_x = -UV_{xx}, \\ V_t + (V(1+U_x))_x = VU_{xx}, \end{bmatrix} \tag{0.2}$$

where units have been renormalized and it is assumed that, in the absence of interaction, the two populations run with the same velocity, -1. The following features are incorporated into this model, where only the total mass of the two populations $\int (U+V)\,dx$ is conserved:

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