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CONVOLUTION OF SET FUNCTIONS

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ABSTRACT. In this paper we study a class of functional operators under which the convexity of convex set functions is preserved. In particular, convolution of convex set functions is defined and its convexity verified.

1. Introduction. The study of set functions has been motivated by recent theoretical results (e.g., [**3**–**8**]) and many applications in different fields (e.g., [**1, 2, 10**]). Since the definition of the convexity for set functions is given in a more general form than that of the ordinary one, it is expected that not all functional operators defined in [**9**; Part I, Section 5] will preserve the convexity of set functions. However, one of the most significant operators, convolution, does preserve the convexity of set functions. The main purpose of this paper is to prove that fact. Some other basic facts concerning the algebra of convex set functions are also explored.

Throughout this paper, it is assumed that $(X, \mathcal{A}, m)$ is an atomless finite measure space with $L_1(X, \mathcal{A}, m)$ separable. For $\Omega \in \mathcal{A}$, χ_Ω denotes the characteristic function of Ω, I the interval $[0, 1]$, and $\overline{\mathbf{R}} = \mathbf{R} \cup \{-\infty, +\infty\}$. We adopt the convention rules as in [**9**; Part I, Section 4] for arithmetic calculations involving $+\infty$ and $-\infty$. In [**8**] Morris showed that for any given Ω, $\Lambda \in \mathcal{A}$ and $\lambda \in I$, there exists a sequence $\{\Gamma_n\} \subset \mathcal{A}$ such that

$$\chi_{\Gamma_n} \xrightarrow{w^*} \lambda\chi_\Omega + (1-\lambda)\chi_\Lambda,$$

where $\xrightarrow{w^*}$ denotes weak* convergence of elements in L_∞. We shall call such a sequence a Morris-sequence associated with $\langle \lambda, \Omega, \Lambda \rangle$. Using Morris-sequence instead of usual convex combinations, a subfamily $\mathcal{S} \subset \mathcal{A}$ is said to be convex if, for every $\langle \lambda, \Omega, \Lambda \rangle \in I \times \mathcal{S} \times \mathcal{S}$ and every Morris-sequence $\{\Gamma_n\}$ associated with $\langle \lambda, \Omega, \Lambda \rangle$ in $\mathcal{S}$, there exists a subsequence $\{\Gamma_{n_k}\}$ of $\{\Gamma_n\}$ in $\mathcal{S}$.

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