

## STABILITY OF A FAKE TOPOLOGICAL HILBERT SPACE

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**ABSTRACT.** The space under consideration is the basic fake Hilbert space  $Y$  of Anderson, Curtis and van Mill. It is shown that the product of an arbitrary space  $A$  with  $Y$  is homeomorphic to  $Y$  if and only if  $A$  is a compact absolute retract. Furthermore, we prove that the complement of  $Y \times Y$  is a capset in  $Q \times Q$ , which implies the known result that  $Y \times Y$  is homeomorphic to Hilbert space.

**1. Introduction.** We are interested in the basic fake Hilbert space  $Y$  that was constructed by Anderson, Curtis and van Mill [1]. The space  $Y$  is the complement of a  $\sigma Z$ -set in the Hilbert cube  $Q$  and, hence, a complete AR. The following properties can be found in [1] and illustrate the closeness of  $Y$  to the Hilbert space  $\ell^2$ : (a)  $Y$  is homogeneous, (b)  $Y \times Y$  is homeomorphic to  $\ell^2$ , and (c)  $Y$  has the weak discrete approximation property. The space has proved to be a very useful basis for the construction of other peculiar spaces and counterexamples as is witnessed by the papers of Anderson et al. [1], Dijkstra and van Mill [8], Dijkstra [7], and Bowers [3]. More information on  $Y$  can be found in Dijkstra [6, Chapters 4 and 5]. The most important results here are the Unknotting Theorem (homeomorphisms between compacta in  $Y$  can be extended with control) and the Negligibility Theorem (the negligible compacta in  $Y$  are precisely the compacta with the shape of a finite set).

In this article we investigate the stability of  $Y$  under multiplication. The result  $Y \times Y \approx \ell^2$  can be improved by showing that the complement of  $Y \times Y$  in  $Q \times Q$  is a capset. We are mainly interested, however, in determining for which spaces  $A$  the product  $Y \times A$  is homeomorphic to  $Y$ . We show that this is the case precisely if  $A$  is a compact absolute retract.

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