ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 22, Number 1, Winter 1992

ON EXTENDED WALLMAN TYPE SPACES

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ABSTRACT. In this paper the usual construction of the Wallman topology on the set of all $(0-1)$-valued, lattice regular measures is extended to the set of all nontrivial, nonnegative, bounded, lattice regular measures. Furthermore, the notion of repleteness is extended to this more general situation.

0. Introduction. In the usual Wallman construction of a compact T_1 space associated with an arbitrary set X and an arbitrary disjunctive lattice of subsets of X, $\mathcal{L}$, one considers the pair $\langle IR(\mathcal{L}), W(\mathcal{L})\rangle$, where $IR(\mathcal{L})$ is the set of $(0-1)$-valued, $\mathcal{L}$-regular measures on $\mathcal{A}(\mathcal{L})$, the algebra of subsets of X generated by $\mathcal{L}$, and $W(\mathcal{L})$ is a certain lattice of subsets of $IR(\mathcal{L})$ (see below for definition). $W(\mathcal{L})$ is then taken as a base for the collection of closed sets of a topology on $IR(\mathcal{L})$, and it turns out that $IR(\mathcal{L})$ with respect to this topology is compact and T_1. (See [**6**].) It is T_2 if and only if $\mathcal{L}$ is normal. If, moreover, $\mathcal{L}$ is separating and X is given the topology with $\mathcal{L}$ as the base for the closed sets, then $IR(\mathcal{L})$ is a compactification of X. Specific cases, where X is a given topological space, give rise to such well-known compactifications of X as ωX, the Wallman compactification of X, βX, the Stone-Čech compactification of X, $\beta_0 X$, the Banaschewski compactification of X, etc.

Considering the set of σ-smooth elements of $IR(\mathcal{L})$, $IR(\sigma,\mathcal{L})$, and the restriction of $W(\mathcal{L})$ to $IR(\sigma,\mathcal{L})$, $W_\sigma(\mathcal{L})$, in [**2**] it was shown that $W_\sigma(\mathcal{L})$ is replete, i.e., for every element of $IR(\sigma,\mathcal{L})$, ν, the support of ν is nonempty. (See also the remark after Theorem 2.4.) If, moreover, $\mathcal{L}$ is separating and X is given the topology mentioned above, then $IR(\sigma,\mathcal{L})$ with the relative topology "contains" X densely, under a suitable

Received by the editors on September 26, 1988, and in revised form on June 26, 1989.
AMS *Mathematics Subject Classification.* Primary 28A60; Secondary 28C15, 54D35.
Key words and phrases. Measure; support of a measure; regularity, σ-smoothness, and τ-smoothness of measures; repleteness, measure repleteness, and support-measure repleteness; premeasure, I-lattice; the Wallman topology.