ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 22, Number 1, Winter 1992

NONINVERTIBILITY OF INVARIANT DIFFERENTIAL OPERATORS ON LIE GROUPS OF POLYNOMIAL GROWTH

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In recent years weighted L^2 spaces have been useful in proving solvability results for invariant differential operators on Lie groups (e.g., [**2, 3**]). This is done by showing that the operators in question are boundedly invertible on a suitable weighted L^2 space.

In this note we present a result which demonstrates some of the limitations of this approach. We show that left invariant differential operators on a connected Lie group, G, of polynomial growth, are not boundedly invertible on $L^2(G, \omega(x)\,dx)$ where dx is the right Haar measure and $\omega(x)$ is a polynomial weight. This should be considered in the context of Levy-Bruhl's use of exponential weights [**2**].

For a measurable subset A of G, let $|A|$ denote the measure of A.

Definition 1. A connected, locally compact group, G, has polynomial growth if there is a polynomial p such that for each compact neighborhood U of e, there is a constant $C(U)$ so that $|U^n| \leq C(U)p(n)$ $(n = 1, 2, \dots)$ $(U^n = \{u_1 \cdot u_2 \cdot \dots \cdot u_n | u_i \in U,\ 1 \leq i \leq n\}.)$ (J. Jenkins has given a characterization of the locally compact groups with polynomial growth in [**1**].)

Note that since G is connected, its growth will be determined by the behavior of $|U^n|$ for any one compact neighborhood U of e.

Definition 2. A nonnegative measurable function ω on a connected Lie group has polynomial growth if there is a polynomial q such that for each compact neighborhood U of e there is a constant $C(U)$ so that

$$\int_{U^n} \omega(x)\,dx \leq C(U)q(n), \qquad n = 1, 2, 3, \dots .$$

Received by the editors on February 22, 1989, and in revised form on April 6, 1989.
Mathematics Subject Classification. Primary 22E30, 58G35.
This research supported in part by a grant from the State of Louisiana.