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ON BEST COAPPROXIMATION IN NORMED LINEAR SPACES

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ABSTRACT. This article is a brief survey of the research concerning best coapproximation, a kind of approximation introduced by C. Franchetti and M. Furi. It is concerned with the existence and uniqueness of elements of best coapproximation by elements of linear subspaces, characterizations of elements of best coapproximation, characterizations of strict convexity in terms of best coapproximation, properties of the best coapproximation operator and best coapproximation on convex sets. Some unsolved and partially solved problems raised by persons working in this field have been mentioned.

1. Introduction. The main object of the theory of best approximation is solution to the following problem: Given a subset G of a normed linear space E and an element $x \in E$, find elements $g_0 \in G$ such that

$$||x - g_0|| \le ||x - g|| \quad \text{for every } g \in G. \tag{1.1}$$

The set of all such elements $g_0 \in G$ (if any) satisfying (1.1) are called elements of best approximation of x by means of the elements of G and is denoted by $P_G(x)$ (see, e.g., [**21**] or [**22**]). Clearly, $P_G(x) = C_G(x) \cap G$, where $C_G(x) = \cap_{g\in G}\bar{b}_{||x-g||}(x), \bar{b}_r(x)$ denotes the closed ball with center x and radius r.

Recently, another kind of approximation from a subspace G, which naturally extends to any set, has been introduced by Franchetti and Furi [**10**], who have considered those elements $g_0 \in G$ (if any) for which

$$||g_0 - g|| \le ||x - g|| \quad \text{for every } g \in G \tag{1.2}$$

and have denoted the set of all such elements $g_0 \in G$ by $R_G(x)$. Any $g_0 \in G$ satisfying (1.2), i.e., any $g_0 \in R_G(x)$, is called an element of "best coapproximation" of x by means of the elements of G. Clearly, $R_G(x) = B_G(x) \cap G$, where $B_G(x) = \cap_{g\in G}\bar{b}_{||x-g||}(g)$.

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