BACKWARD EXTENSIONS AND
STRONG HAMBURGER MOMENT SEQUENCES

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ABSTRACT. Strong Hamburger moment sequences are studied. Necessary and sufficient conditions for the strong Hamburger moment problem to have a unique solution are obtained. These results are based on certain of the solutions to the classical Hamburger moment problem. The nested disks and orthogonal polynomials associated with the corresponding Jacobi type continued fraction are used.

1. Introduction. A double sequence of real numbers $\{c_n : n = 0, \pm 1, \pm 2, \dots\}$ is called a strong Hamburger sequence (SHMS) if there is a bounded nondecreasing function ϕ on the interval $-\infty < t < \infty$ such that

$$(1) \qquad\qquad c_n = \int_{-\infty}^{\infty} t^n \, d\phi(t)$$

for all integers n. A real sequence $\{c_n : n = 0, 1, 2, \dots\}$ is a (classical) Hamburger moment sequence (HMS) if there is a bounded nondecreasing ϕ such that (1) holds for all nonnegative integers n. In each case, the function ϕ is called a distribution function for the given sequence.

Extensive work has been done in recent years on the strong moment problems. In particular, we mention the work of Jones, Thron, and Waadeland [7] on a strong Stieltjes moment sequence (a Hamburger moment sequence in which the distribution function ϕ is constant on $(-\infty, 0)$). In their study, they emphasized continued fraction methods, specifically T-fractions. More recently, Jones, Njåstad, and Thron [4,5] and Njåstad and Thron [9] have investigated strong Hamburger moment sequences. Their studies use several methods, including continued fractions, orthogonal polynomials, and nested disks. Good historical sketches of moment problems can be found in the works of Jones and Thron [6] and [9].