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DENSITY AND THE CIRCULAR PROJECTION

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0. Introduction. One aspect of complex analysis deals with classifying the points $a \in \partial D$, D a simply connected domain, by determining whether a certain geometric condition exists or fails to exist in a neighborhood of a. See [**1, 3, 4**]. These geometric conditions sometimes indicate how some part of the boundary of D, say $E \subset \partial D$, geometrically behaves near a. See [**2**]. When some sort of an inner normal at $a \in \partial D$ exists, one can define a set S on the normal that is the image of E under a circular projection. If the set S happens to have certain density properties, does E have them also? We answer this question in a certain setting.

In Section I we discuss the basic definitions and properties of density on the real number line and on a rectifiable Jordan arc. We then consider the curve $\Gamma : y = f(x)$, $0 \leq x \leq m$, where $f(x)$ satisfies a Lipschitz condition and show that $(x_0, f(x_0))$ is a point of density of a measurable set B of Γ if and only if x_0 is a point of density of $P(B)$, where P is the projection map $P : \Gamma \to [0, m]$. Section 2 considers a point $a_0 \in \Gamma$ where the inner normal exists, defines the circular projections C_R, C_L from Γ into the inner normal at a_0, and shows that with a Lipschitz constant less than one a similar result holds for C_R and C_L. Finally, Section 3 shows that if the Lipschitz constant is greater than one, then the theorem is true in one direction for C_R, C_L but not in the other.

1. Density and projections. We begin our discussion of density on the real number line which we denote by $\mathbf{R}$. Let m denote Lebesgue measure and m^* the outer measure with respect to m. We shall say that *a sequence* $\{I_k\}$ *of intervals in* $\mathbf{R}$ *converges to* $x \in \mathbf{R}$ and write $I_k \to x$, $x \in \mathbf{R}$, if $x \in I_k$ for each k and $\lim_{k\to\infty} \operatorname{diam} I_k = 0$.

Let A be any subset of $\mathbf{R}$. For any measurable set E in $\mathbf{R}$ we define

$$\sigma_A(E) = m^*(A \cap E).$$

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