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THE BOUNDARY BEHAVIOR OF THE KOBAYASHI METRIC

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1. Introduction. Let Ω be a *domain*, that is, a connected open set, in $\mathbf{C}^n$. If $P \in \Omega$ and $\xi \in \mathbf{C}^n$, then define $\mathcal{M}(P,\xi)$ to be the collection of holomorphic mappings ϕ of the unit disc D into Ω such that $\phi(0) = P$ and $\phi'(0)$ is a scalar multiple of ξ. The *Kobayashi* (or *Kobayashi/Royden*) *length* of ξ at the point P is defined to be

$$F_K^\Omega(P,\xi) \equiv \inf\{\alpha : \alpha > 0, \exists \phi \in \mathcal{M}(P,\xi) \text{ with } \phi'(0) = \xi/\alpha\}.$$

See [**6**] and [**5**] for more on the Kobayashi metric.

This metric is becoming increasingly important in the function theory of several complex variables (see [**7, 8, 9, 10, 11, 12**]). In particular, it is important to calculate and estimate the metric on a variety of domains. An interesting conjecture is that any smoothly bounded pseudoconvex domain is complete in the Kobayashi metric.

If $P \in \Omega$, then let $\delta(P) = \delta_\Omega(P)$ denote the distance of P to $\partial\Omega$. An important step in determining the validity of the last conjecture would be to prove that, for P near the boundary and $\xi = \nu_P$ (the unit outward normal vector to $\partial\Omega$ at P) it holds that

$$F_K^\Omega(P,\xi) \approx \frac{1}{\delta(P)}.$$

Here the notation $A \approx B$ means that the quotient A/B is bounded above and below by absolute constants.

The mapping

$$\phi(\zeta) = P + \delta(P)\zeta\nu_P,$$

together with the definition of the metric, shows that

$$F_K^\Omega(P,\xi) \le C \cdot \frac{1}{\delta(P)}.$$

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