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ALGEBRAS WITH THE LOCAL INTERPOLATION PROPERTY

Ma TERESA GASSÓ

ABSTRACT. In this paper a class of Boolean algebras is defined in such a way that the classical Nikodym theorem holds for sequences of bounded additive measures defined on said algebras. It is proved that this class of Boolean algebras contains those known to have the property (N), i.e., the ones satisfying the Vitali-Hahn-Saks theorem [**10**] as well as those introduced by Schachermayer [**9**] and by Graves-Wheeler [**5**].

The second problem raised by Graves and Wheeler in [**5**] is solved because the local interpolation (LI) alone proves the property (N). The condition (LI) gives a new example of Boolean algebras with the Nikodym property.

The Boolean algebras of Seever [**10**] and Faires [**3**] and those studied here are defined by means of "interpolation properties" between disjoint sequences in this algebra.

1. Introduction. The book by Diestel and Uhl [**2**] gives us an account of the history of Grothendieck, Nikodym and Vitali-Hahn-Saks properties. We must remember that Diestel, Faires and Huff [**1**] proved that a Boolean algebra has the property (VHS) if and only if it has the property (N) and the space of Banach of real and continuous functions on the Stone space of the algebra is a Grothendieck space, (property (G)) [**6**]. A characterization of the algebras with the property (G) would be interesting in the isomorphic classification of the Banach spaces. Such characterization was conjectured by Lindenstrauss [**7**] but has not yet been satisfactorily solved. Once the equivalence between (VHS) and (G)–(N) and the question of Lindenstrauss are established, the implications (N) $\Rightarrow$ (G)? and (G) $\Rightarrow$ (N)? are naturally raised. In fact, the first of these questions appears in [**10**] and was posed by Seever. This problem was solved by Schachermayer [**9**] who shows that (N) $\not\Rightarrow$ (G) by means of two examples. We want to note that the proof of (N) is different in each case. In J_1, the algebra of Jordan measurable subsets of [0,1], the property (N) is inferred from the compactivity in [0,1], whereas in the other example J_2, it is proved directly in the algebra.

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