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A SKOROHOD REPRESENTATION AND AN INVARIANCE PRINCIPLE FOR SUMS OF WEIGHTED i.i.d. RANDOM VARIABLES

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ABSTRACT. A Skorohod representation is obtained for sums of weighted i.i.d. random variables, extending the i.i.d. case. This leads to a functional law of the iterated logarithm and other invariance results. In this setting, the results are not included as special cases of previous martingale results.

1. Introduction. Let $\{X_k : k = 1, 2, \dots\}$ be a sequence of i.i.d. random variables with $EX_1 = 0$ and $EX_1^2 = 1$. Let $\{a_k : k = 1, 2, \dots\}$ be a sequence of real numbers. We refer to these as "weights." Define the sum, S_n, of weighted i.i.d. random variables as $S_n = \sum_{k=1}^n a_k X_k$.

In Section 2 of this paper a Skorohod representation is obtained for the sums S_n. This is the content of Theorem 2.1 and Theorem 2.2. These results extend the original representation by Skorohod [**11**] for sums of i.i.d. random variables.

Section 3 consists of applications of the Skorohod representation derived in Section 2. In particular, we obtain a functional law of the iterated logarithm (Theorem 3.2) and an almost sure invariance principle for sums of weighted i.i.d. random variables (Theorem 3.3). These are analogous to the results obtained by Strassen [**13**] in the i.i.d. case. A central limit theorem (Theorem 3.1) and a classical law of the iterated logarithm (Corollary 3.4) are also obtained. We remark that all the results derived here are extensions of the i.i.d. case, i.e., where $a_k \equiv 1$.

Since Skorohod and Strassen proved their results for i.i.d. random variables, analogous results have been obtained for martingales, notably by Strassen [**14**], Jain, Jogdeo and Stout [**7**], Heyde and Scott [**6**], and Hall and Heyde [**5**]. However, the Skorohod representation derived in

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