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AN EULERIAN METHOD FOR REPRESENTING π^2 BY SERIES

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ABSTRACT. Following a method of Euler, the author presents three apparently new series representations of π^2.

1. Introduction. Leonhard Euler was so impressed by his now famous formula

$$\frac{\pi^2}{6} = \sum_{n=1}^{\infty} \frac{1}{n^2} \tag{1}$$

that he offered several different proofs of it. Among these methods of proof we wish to highlight a certain one, which according to G. Turnwald [**4**, p. 331] was rediscovered by Boo Rim Choe [**2**, p. 662–663] some 244 years later. This method is briefly described as follows:

(i) For $0 \le x < 1$, first evaluate the integral

$$\int_0^x \frac{dt}{\sqrt{1-t^2}}$$

directly, and then evaluate it by expanding the integrand, followed by termwise integration, to get

$$\sin^{-1} x = \sum_{n=0}^{\infty} \frac{\binom{2n}{n}}{2^{2n}} \frac{x^{2n+1}}{2n+1}. \tag{2}$$

(ii) In (2) let $x \to \sin x$, integrate the resulting equation from 0 to $\pi/2$, and simplify by appeal to Wallis's formula in the form

$$\int_0^{\pi/2} \sin^{2k+1} x\, dx = \frac{1}{2k+1} \frac{2^{2k}}{\binom{2k}{k}}, \qquad k \ge 0, \tag{3}$$

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