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CLASSIFYING GENERIC ALGEBRAS

TH. DANA-PICARD AND M. SCHAPS

ABSTRACT. This paper proposes a program for the inductive classification of the generic unitary associative algebras of dimension n, based on previous work in the field and on two new results pertaining to different aspects of the problem. The first is a diagonalization theorem, showing that the nonnilpotent sections of an idempotent-creating deformation can be chosen from within the direct sum of the local rings at the newly created idempotents. The second gives sufficient conditions for a "loopless" basis graph with a specified radical flag structure to determine a unique component of the structure-constant scheme Alg_n. The procedure for classifying generic algebras is then described and illustrated by determining the generic algebras of dimension six.

1. Introduction. The classical problem of classifying n-dimensional algebras suffers from being too easy. Once the ground rules are explained, a competent algebraist with time and patience can sit down and generate multiplication tables for associative algebras, but the activity becomes unilluminating around dimension six and has not been carried much further. Such calculations flourished for a while at the end of the last century [**12**] but more or less died out in the face of more general structure theorems, particularly the Wedderburn theorems.

The subject became more interesting when it was broadened to include determining not only the algebras themselves but also the partial ordering of the algebras by the relation of specialization. Of particular interest from this point of view are the generic algebras, the maximal algebras or families of algebras with regard to this partial ordering.

An n-dimensional algebra is generally defined by fixing a K-basis $v_1, \dots, v_n$ and giving the multiplication structure

$$v_i v_j = \sum_k a_{ij}^k v_k. \tag{1}$$

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