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REPRESENTATION OF THE ATTAINABLE SET FOR LIPSCHITZIAN DIFFERENTIAL INCLUSIONS

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1. Introduction. In this paper we consider the Cauchy problem

$$\text{(CP)} \qquad x' \in F(t,x), \qquad x(0) = \xi,$$

where F is Lipschitzian with respect to x, with values that are closed (not necessarily convex nor bounded) subsets of $\mathbf{R}^n$ and ξ ranges in a compact subset Ξ of $\mathbf{R}^n$. We show that the map that assigns to each ξ the set of solutions of (CP), $S(\xi)$, can be continuously represented as

$$S(\xi) = g(\xi, \mathcal{U}).$$

The same result holds for the map from ξ to the attainable set at time T, $\mathcal{A}_T(\xi)$, which in general is not a closed set. Similar representations of set valued maps were known in case the values are compact convex; see [**3, 7, 8**].

In order to obtain our representation, we prove first a continuous selection theorem from the map $S(\xi)$, which is more precise than the result presented in [**2**]. Moreover, we do not assume the boundedness of the values of F, and our proof is considerably simpler than the proof in [**2**]. In particular, we do not need either Liapunov's theorem on the range of a vector measure or any previous existence result.

2. Notation and preliminary results. In what follows we denote by $\operatorname{dl}(A,B)$ the Hausdorff distance between the sets $A, B \subset \mathbf{R}^n$ (see [**6**]). The distance of a point x from a set A, $d(x,A)$, is $\inf\{|x-a| : a \in A\}$. I is the interval $[0,T]$; the characteristic function of a subset E of I is χ_E. We consider AC the space of absolutely continuous functions from I to $\mathbf{R}^n$ with norm $||x||_{AC} := |x(0)| + \int_0^T |x'(\tau)|\, d\tau$. We assume

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