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A BASIC CONSTRUCTION IN DUALS OF SEPARABLE BANACH SPACES

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ABSTRACT. A basic construction of the Cantor set Δ in the dual of a separable Banach space X is presented. If X^* is nonseparable, a modification of this construction yields bounded ε-trees in X^* (Stegall). A continuous linear surjection from X to $C(\Delta)$ is obtained if ℓ^1 embeds in X (Pelczynski) by a further modification of this construction. Through it the delicate nature of the difference between the cases (i) X^* is nonseparable and (ii) ℓ^1 embeds in X is highlighted.

A. Introduction. Let Δ^0 denote the usual Cantor set with dyadic partitions $(C_{ni}^0 : i = 1, \dots, 2^n)_{n=0}^\infty$ and Haar measure λ^0 (where $\lambda^0(C_{ni}^0) = 2^{-n}$ for all i and n). Let $\lambda_{ni}^0(\cdot) = 2^n \lambda^0((\cdot) \cap C_{ni}^0)$.

Now let Δ denote the natural copy of Δ^0 in $C(\Delta^0)^*$, the points of Δ corresponding to point-masses on $C(\Delta^0)$. Let λ_{ni} denote λ_{ni}^0 as a measure on Δ. We think of λ_{ni}^0 in $C(\Delta^0)^*$ as the barycenter of the measure λ_{ni} on Δ. Note that the λ_{ni}^0's form a bounded ε-tree, with $\varepsilon = 2$, as $\lambda_{ni}^0 = (1/2)\ (\lambda_{n+12i-1}^0 + \lambda_{n+1,2i}^0)$ and $||\lambda_{n+1,2i-1}^0 - \lambda_{n+1,2i}^0|| = 2$.

Now suppose X is a separable Banach space and X^* is nonseparable. Then it is easy (see Corollary 2 below) to construct a topological copy of Δ in (B^*, weak^*) which is norm discrete (and conversely the existence of such a set obviously implies X^* is nonseparable). C. Stegall [**7**] showed how to construct such a Δ and corresponding dyadic partitions (C_{ni}), with Haar measure λ, so that the barycenters x_{ni}^* of the measures $\lambda_{ni}(\cdot) = 2^n \lambda((\cdot) \cap C_{ni})$ on Δ form a bounded ε-tree in X^*.

On the other hand, the Pelczynski–Hagler theorem states that ℓ^1 embeds in a separable Banach space X if and only if there exists a continuous linear surjection from X to $C(\Delta^0)$ [**3, 4**]. In this paper a basic construction is presented which obtains these two results and highlights the delicate differences between them.

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