ON CONNECTED GROUPS AND RELATED TOPICS

R.W. BAGLEY, T.S. WU, AND J.S. YANG

ABSTRACT. The study of pro-Lie groups and residual Lie groups [**2, 3**] led to the present work on connected groups and related topics. During the course of our study, we found that some theorems (Theorem 8, for example) of V.M. Gluskov as appeared in [**5**] may not have been accurately stated. In a related context we show that if G is a connected, locally connected, locally compact group with its center metrizable, then there exists a neighborhood of the identity element each of whose points lies in a one parameter subgroup. In terms of the characteristic index of a connected locally compact group defined by K. Iwasawa [**8**], we have introduced the concept of total index for a closed normal subgroup of a connected locally compact group. A counterexample to a conjecture made in [**6**] is given at the end of the paper.

1. Introduction. This paper constitutes part of our recent efforts in studying the structure of locally compact groups. Our study of pro-Lie groups and residual Lie groups [**2, 3**] led to the present work.

In Section 2 we study some properties of connected locally compact groups and the question of lifting one-parameter subgroups. We show that the arc component of the identity element of a connected locally compact group is generated by all one parameter subgroups. During this course of our study, we found that some theorems (Theorem 8, for example) of V.M. Gluskov as appeared in [**5**] may not have been accurately stated. In a related context we show that if G is a connected, locally connected, locally compact group with its center metrizable, then there exists a neighborhood of the identity element each of whose points lies in a one-parameter subgroup. We also study local connectedness of homogeneous spaces at the end of the section.

If G is a connected locally compact group, then there exists a maximal compact connected subgroup K such that $G = KE$, where the space E is homeomorphic to γ-dimensional Euclidean space. The integer γ