ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 22, Number 1, Winter 1992

ON THE DIOPHANTINE EQUATION $1 + x + y = z$

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ABSTRACT. In this paper all solutions to the equation $1 + x + y = z$, where x, y and z are positive integers such that xyz has the form $2^r 3^s 5^t$, with r, s and t nonnegative integers, are determined. This work extends earlier work of the authors and J.L. Brenner in the field of exponential Diophantine equations.

1. Introduction. In this paper we consider the equation

$$1 + x + y = z, \tag{1.1}$$

where x, y and z are positive integers such that xyz has the form $2^r 3^s 5^t$, for nonnegative integers r, s and t. This equation has the form

$$\Sigma x_i = 0, \tag{1.2}$$

where the primes dividing Πx_i are specified.

There has been little work done in general to solve such Diophantine equations. Some of these equations have an infinite number of trivial solutions. For example, the still-unsolved equation

$$1 + 2^a 3^b = 5^c + 2^d 3^e 5^f$$

has infinitely many solutions of the form $c = f = 0$, $a = d$, $b = e$. It is unknown whether such equations must have only a finite number of nontrivial solutions.

It follows from the work of Dubois and Rhin [**7**] and Schlickewei [**8**] that the related equation $p^a \pm q^b \pm r^c \pm s^d = 0$ has only finitely many solutions when p, q, r and s are distinct primes. However, their methods do not seem to apply when the terms in the equation are not powers of distinct primes.

The authors and J.L. Brenner [**1, 2, 4-6**] have recently developed techniques which solve such equations in some cases. These techniques

Received by the editors on March 5, 1986.