ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 22, Number 1, Winter 1992

ON THE GROTHENDIECK AND NIKODYM PROPERTIES OF BOOLEAN ALGEBRAS

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ABSTRACT. In this paper we investigate the Grothendieck and Nikodym properties on a Boolean algebra. We obtain a sufficient condition for the Nikodym property that is not sufficient for the Grothendieck property.

1. Introduction. Schachermayer [**9**] proved that the family J of Jordan-measurable sets in [0,1] has the Nikodym property (NP) but lacks the Grothendieck property (GP). This is the first algebra known with these characteristics and it has been of major importance in the study of the relations between the Grothendieck and Nikodym properties (cf. [**3**]). Graves and Wheeler [**5**] made another important contribution to this subject, the main focus of which is the study of the properties (NP) and (GP) for certain Jordan-type algebras of Baire, Borel and universally measurable sets.

Haydon [**6**] proved that a subsequentially complete Boolean algebra (SC) has the Grothendieck property. He also gives an example of a Boolean algebra with the property (SC) that does not have Rosenthal's property. Hence, he obtains a sufficient condition for Grothendieck's property that is not sufficient for Rosenthal's property.

Dashiell [**2**] proved that an algebra that is up-down-semi-complete (udsc) and that has an additional property, has Rosenthal's and Nikodym's properties. Hence, such algebra has Grothendieck's property. An algebra with that additional property will be called (aD).

Dashiell [**2**] also proved that the Boolean algebra D of the simultaneously G_δ and F_σ sets in [0,1] has the properties (udsc) and (aD). We observe that the algebra J is (udsc) and, hence, the property (aD) gives to D properties that J does not have.

Talagrand [**10**] proves that, assuming the continuum hypothesis, there exists a Boolean algebra with the Grothendieck property that lacks the Nikodym property.

Received by the editors on November 5, 1988 and in revised form on August 14, 1989.