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EXTENSION OF TOPOLOGICAL INVARIANT MEANS ON A LOCALLY COMPACT AMENABLE GROUP

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1. Introduction. Let G be a locally compact group associated with its left Haar measure. For a Borel set $A \subset G$ we denote by $|A|$ the measure of A. Let $L^\infty(G)$ be the Banach space of all essentially bounded Borel measurable functions on G, and let $CB(G)$ be that of all bounded continuous functions. For a function f in $CB(G)$, we say that f is left uniformly continuous if, given $\varepsilon > 0$, there is a neighborhood U of e, the identity in G, such that

$$|f(x) - f(xy)| < \varepsilon, \quad x \in G, \ y \in U.$$

The space of all left uniformly continuous bounded functions is denoted by $UCB_l(G)$. The right uniform continuity and the space $UCB_r(G)$ are defined symmetrically. The space of all uniformly continuous bounded functions is defined by $UCB(G) = UCB_l(G) \cap UCB_r(G)$. These spaces are all considered as subspaces of $L^\infty(G)$, with the supremum norm $||\cdot||_\infty$.

For each $x \in G$ and $f \in L^\infty(G)$, we define a new function ${}_xf \in L^\infty(G)$ by ${}_xf(t) = f(x^{-1}t)$ for all $t \in G$. For a closed subspace X of $L^\infty(G)$, we say that X is (left) translation invariant if $f \in X$ implies that ${}_xf \in X$ for all $x \in G$. Each of the above spaces is (two sided) translation invariant. For a translation invariant space X containing $UCB(G)$, we define a left invariant mean μ on X to be a positive element in X^* ($\mu(f) \geq 0$ if $f \in X$ is nonnegative) of norm 1 such that $\mu({}_xf) = \mu(f)$ for all f in X and $x \in G$. The existence of an invariant mean on $L^\infty(G)$, or on $UCB(G)$, or on any intermediate space is equivalent. If G admits an invariant mean on any of these spaces, we say G is amenable. Let G_d be the same algebraic group as G with a discrete topological structure. If G_d admits a left invariant

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