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AN EXTENSION OF ASKEY-WILSON'S q-BETA INTEGRAL AND ITS APPLICATIONS

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1. One of the remarkable q-extensions of the classical beta integral evaluated by Askey and Wilson [**3**] is:

If $\max(|a|,|b|,|c|,|d|)<1$, then

$$\begin{aligned}
&\int_{-1}^{1} w(z;a,b,c,d)\,dz = K, \text{ where}\\
&w(z;a,b,c,d)=\frac{h(z;1)h(z;-1)h(z;\sqrt{q})h(z;-\sqrt{q})}{h(z;a)h(z;b)h(z;c)h(z;d)\sqrt{1-z^2}},\\
&h(z;a)=\prod_{n=0}^{\infty}(1-2azq^n+q^{2n})\\
&\qquad=(ae^{i\theta},ae^{-i\theta};q)_\infty, \qquad z=\cos\theta,\\
&(a;q)_\infty=\prod_{j=0}^{\infty}(1-aq^j), \text{ whenever it converges,}\\
&(a_1,a_2,\dots,a_n;q)_\infty=(a_1;q)_\infty\dots(a_n;q)_\infty
\end{aligned} \tag{1.1}$$

and

$$K=\frac{2\pi(abcd;q)_\infty}{(q,ab,ac,ad,bc,bd,cd;q)_\infty}.$$

Nassrallah and Rahman [**6**] used (1.1) to obtain q-analogues of Euler's integral representation of Gauss's hypergeometric series ${}_2F_1$:

$$\begin{aligned}
&\int_{-1}^{1} w(z;a,b,c,d)\frac{h(z;\lambda)}{h(z;f)}\,dz\\
&\quad=K\frac{(\lambda a,\lambda b,\lambda c,abcf;q)_\infty}{(af,bf,cf,\lambda abc;q)_\infty}\cdot {}_8W_7\left[\frac{\lambda abc}{q};bc,ac,ab,\lambda/d,\lambda/f;q,df\right],
\end{aligned} \tag{1.2}$$

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