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UNBOUNDED MULTIVALUED NEMYTSKII OPERATORS IN SOBOLEV SPACES AND THEIR APPLICATIONS TO DISCONTINUOUS NONLINEARITY

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Introduction. The aim of this paper is to provide tools for solving boundary value problems for partial differential inclusions of the type

$$\mathrm{L}u \in F(x, u, Du, \dots), \qquad x \in \Omega,$$

where the order of a linear operator L exceeds the order of a Carathéodory multifunction F with convex values in R^M. Such inclusions may be used for dealing with partial differential equations with discontinuous nonlinearities, as we illustrate by an example at the end of this paper.

In all publications on that topic known to the author, F was assumed as either bounded or growing at most linearly in function variables (cf. [**4, 5, 13, 15**]). However, if $\dim \Omega = 1$, such assumptions can be relaxed, as shown, e.g., in [**9**] or [**11**]. We establish here results (Theorem 1 and Corollary 1) showing that the linear growth need not be assumed with respect to those partial derivatives of u which, due to the Sobolev imbedding theorem, imbed to C.

The next presented results (Lemma 2 and Theorem 3) show how solutions of differential inclusions can be approximated by solutions of equations

$$\mathrm{L}u = f_\varepsilon(x, u, Du, \dots), \qquad x \in \Omega,$$

where f_ε is a Carathéodory function whose graph approximates the graph of F. The consequences of Theorem 3 are twofold; on one hand, it provides a theoretical interpretation of a common practice of ignoring continuity assumptions while using numerical methods in differential equations. On the other hand, we hope to use Theorem 3 for directly deriving existence results for partial differential inclusions from the corresponding results for partial differential equations without repeating the same arguments for multivalued operators.

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