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ALMOST COMPLETELY DECOMPOSABLE TORSION-FREE GROUPS

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Recently, D. Arnold and C. Vinsonhaler were successful in developing a complete set of numerical invariants for certain pure subgroups and homomorphic images of completely decomposable abelian groups [**4**]. Their results extended work by F. Richman [**8**]. Let A_i be subgroups of the rationals, $\mathbf{Q}$, containing the integers, $\mathbf{Z}$. The groups that they consider are the strongly indecomposable groups of the form: $A_1 \oplus \cdots \oplus A_n / \langle(1,1,\dots,1)\rangle_*$; or $\operatorname{Ker}\sigma$, where $\sigma : A_1 \oplus \cdots \oplus A_n \to \mathbf{Q}$ is defined by $\sigma(a_1,\dots,a_n) = \sum a_i$.

In this paper we assume that A is almost completely decomposable (quasi-isomorphic to a finite direct sum of subgroups of $\mathbf{Q}$), and give conditions on the types of the quasi-summands of A which characterize when every pure subgroup and/or every torsion-free homomorphic image of A is again almost completely decomposable. Our results generalize a theorem of M.C.R. Butler [**6**].

Throughout, all groups considered are torsion-free abelian. If n is a positive integer ≥ 2, we write $\bar{n}$ for the set $\{1,2,\dots,n\}$. Suppose for each $i \in \bar{n}$, τ_i is a type. If $I \subseteq \bar{n}$ is nonempty, we write τ^I (respectively, τ_I) for $\sup\{\tau_i : i \in I\}$ (respectively, $\inf\{\tau_i : i \in I\}$). If $I = \{i,j\}$, we often write τ^{ij} or $\tau_i \vee \tau_j$ (respectively, τ_{ij} or $\tau_i \wedge \tau_j$) for τ^I (respectively, τ_I).

1. Almost completely decomposable homomorphic images.

We will assume that $A = A_1 \oplus \cdots \oplus A_n$ with $\mathbf{Z} \leq A_i \leq \mathbf{Q}$. Set $G\langle A\rangle = A/X$, where X is the pure subgroup generated by $(1,1,\dots,1)$.

Let $f : A_1 \oplus \cdots \oplus A_n \to G\langle A\rangle$ be the natural quotient map. If K is corank-1 in $G\langle A\rangle$ (i.e., if $G\langle A\rangle/K$ is rank-1 and torsion-free), set $\operatorname{cosupp}(K) = \{i \in \bar{n} : f(A_i) \leq K\}$. By [**2**, Theorem 1.4], there is a cobalanced embedding $\delta : G\langle A\rangle \to \oplus\{G\langle A\rangle/K : K$ is corank-1 and $\operatorname{cosupp}(K)$ is maximal with respect to inclusion$\}$, where δ is induced

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