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ASYMPTOTIC ANALYSIS OF QUENCHING PROBLEMS

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Introduction. Let $\Omega \subset \mathbf{R}^n$ be a bounded domain with smooth boundary and let $\alpha > 0$. Consider the problem

$$(P)\begin{cases} u_t - \Delta u = -u^{-\alpha} & \text{in } (0,T)\times\Omega,\\ u = 1 & \text{on } (0,T)\times\partial\Omega,\\ u(0,x) = u_0(x) & \text{in } \Omega.\end{cases}$$

Here $0 < u_0(x) \le 1$ is assumed throughout the paper. It is well known that for sufficiently large domains Ω the solution can approach zero in finite time, see [**10,2**]. This phenomenon is called quenching and throughout this paper we assume that u quenches at time $T < \infty$.

It was furthermore shown that $u_t \to -\infty$ as $u \to 0$, see [**10,5,1,6**].

In the present paper we derive some asymptotic estimates for u near the point $(T, 0)$ in which u is supposed to quench. They will be of the type

$$\min_{x\in\Omega} u(t,x) \le [(1+\alpha)(T-t)]^{1/(1+\alpha)}, \tag{1}$$

$$u(t,x) \ge C_1(T-t)^{1/(1+\alpha)}, \tag{2}$$

$$u(T,r) \le C_2 r^{2/(1+\alpha)} \qquad \text{for } \alpha < 1, \tag{3}$$

$$u(t,r) \ge C_3 r^{2/(1+\gamma)} \qquad \text{for } 0 < \gamma < \alpha. \tag{4}$$

Notice that (2) implies the blow up of u_t at quenching. Therefore, (1) and (2) give us the rate at which $u_t \to -\infty$. Notice further that (3) implies $u_r(T,0) = 0$ for $0 < \alpha < 1$, see also Remark 2.10. (3) and (4) will be derived only in a radial situation where Ω is a ball and u_0 radially symmetric and for (3) $u_0 \equiv 1$. A consequence of (4) is the fact that for $n \ge 2$ or $\alpha < 3$

$$\|1 - u(t,\cdot)\|_{H_0^1(\Omega)} \le C_4 \quad \text{for any } t \in (0,T),$$

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