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EXISTENCE OF SOLUTIONS TO BOUNDARY VALUE PROBLEMS FOR IMPULSIVE SECOND ORDER DIFFERENTIAL INCLUSIONS

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ABSTRACT. We consider nonlinear boundary value problems for second order differential inclusions $y'' \in F(t, y, y')$ where the solution undergoes an impulse at certain points t_k. The technique used is an adaptation of the topological transversality method to systems with impulses.

1. Introduction. In this paper we shall study the following boundary value problem for a system of second order impulsive differential inclusions:

$$
(1.1) \qquad \begin{cases} y'' \in F(t, y, y') & \text{for a.e. } t \in [a_0, a_1] \\ y(t_k^+) = I_k(y(t_k)) & \\ y'(t_k^+) = N_k(y(t_k), y'(t_k)) & k = 1, \dots, m \\ G_i(\tilde{y}) = 0 & i = 0, 1, \end{cases}
$$

where

(i) $F : [a_0, a_1] \times \mathbf{R}^n \times \mathbf{R}^n \to 2^{\mathbf{R}^n}$ is a multifunction,

(ii) $I_k : \mathbf{R}^n \to \mathbf{R}^n$ is a homeomorphism for $k = 1, \dots, m$,

(iii) $N_k : \mathbf{R}^n \times \mathbf{R}^n \to \mathbf{R}^n$ is continuous, $k = 1, \dots, m$

(iv) $\tilde{y} = (y(a_0), y'(a_0), y(a_1), y'(a_1))$ and

$$G_i : \mathbf{R}^{4n} \longrightarrow \mathbf{R}^n \text{ is continuous, } i = 0, 1.$$

(v) $a_0 = t_0 < t_1 < \dots < t_m < t_{m+1} = a_1$.

We look for a solution to (1.1) which is a piecewise C^1-function with points t_k of discontinuity, $k = 1, \dots, m$, of the first type for y and y' at which they are left continuous.

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