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DUALITY IN SOME VECTOR-VALUED FUNCTION SPACES

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ABSTRACT. We prove two results concerning duality in some function spaces. First we show that for $1 \leq p \leq \infty$ and X a complex Banach space, the space $H^p(D, X^*)$ is isometrically isomorphic to a dual space and we use this result to get a characterization of the analytic Radon-Nikodym property in dual spaces. Second, we show that if Λ is an infinite Sidon subset of the dual of a compact abelian metrizable group, if X is a Banach space and $1 \leq p \leq \infty$, then $L^p_\Lambda(G, X^*)$ is a dual space if and only if X^* does not contain a copy of c_0.

1. Introduction. In [**3**] Bochner and Taylor proved that if $1 \leq p < \infty$, $1/p + 1/q = 1$ and X is a Banach space, then $(L^p([0,1]; X))^* = L^q([0,1]; X^*)$ if and only if X^* has the Radon-Nikodym property with respect to Lebesgue measure on $[0,1]$. They also gave a representation of $(L^p([0,1]; X))^*$ when $1 \leq p < \infty$ and X is any Banach space. In this note we make use of this representation in two settings. In Section 2 we will show that $H^p(D, X^*)$ is a dual space where X is a Banach space and $1 \leq p \leq \infty$. As an application we obtain a new characterization of the analytic Radon-Nikodym property in dual spaces. In Section 3, we consider the function space $L^p_\Lambda(G, X^*)$, where G is a compact abelian metrizable group, Λ is a Sidon subset of the dual group of G and X is a Banach space. We show that $L^p_\Lambda(G, X^*)$ is a dual space for $1 \leq p \leq \infty$, if and only if X^* does not contain a copy of c_0.

2. The analytic Radon-Nikodym property. We denote by $(\Pi, \mathcal{B}, m)$ the Lebesgue space on the unit circle Π with $m(\Pi) = 1$ and D will denote the open unit disk in the complex plane.

Let X be a complex Banach space and let $1 \leq p \leq \infty$. The space $H^p(D, X)$ consists of all holomorphic functions $f : D \to X$ satisfying

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