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RIMCOMPACTNESS AND SIMILAR PROPERTIES IN PREIMAGES

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ABSTRACT. It is shown that the properties of rimcompactness, almost rimcompactness and having a compactification with totally disconnected remainder are preserved in preimages under closed maps whose point preimages have the appropriate property and have compact zero-dimensional boundary. Examples indicate that the hypotheses on the maps cannot be weakened significantly.

All spaces in this paper will be completely regular and Hausdorff. A space X is *rimcompact* if X has a base of open sets with compact boundaries, *almost rimcompact* if X has a compactification KX in which each point of the remainder $KX\backslash X$ has a base of open sets of KX whose boundaries lie in X, and *TDI* (for totally disconnected at infinity) if X has a compactification with totally disconnected remainder. Note that an almost rimcompact space is TDI.

Various authors have considered the preservation of the above properties in images and preimages, with an interesting duality developing. "Map" will mean continuous surjection; a map $f : X \to Y$ is *perfect* (*rimperfect*, respectively) if f is closed and $f^{\leftarrow}(y)$ ($bd_x f^{\leftarrow}(y)$ respectively) is compact for $y \in Y$, and *monotone* if $f^{\leftarrow}(y)$ is connected for $y \in Y$. It is shown in [**1**] that rimcompactness is preserved in images under rimperfect monotone maps. Following on this work, we showed in [**3**] that the properties of almost rimcompactness and having a compactification with 0-dimensional remainder are preserved in rimperfect monotone images. Also, the properties of rimcompactness and almost rimcompactness are preserved in preimages of such spaces under rimperfect maps whose point preimages are zero-dimensional. Since the perfect monotone preimage of a rimcompact space need not be rimcompact [**4**], zero-dimensionality in some form is important in this last result. In this paper we prove the stronger result stated in the abstract.

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