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ON THE REPRESENTATION OF MEASURABLE SET VALUED MAPS THROUGH SELECTIONS

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1. Introduction. Under reasonable hypotheses, a measurable multi-function F admits measurable selections. Actually [**1**], in this case, one can describe the whole multi-function through a countable family $\mathcal{F}$ of selections, in the sense that

$$F(x) = \mathrm{cl}\,\{f(x) : f \in \mathcal{F}\}.$$

In the present paper we consider an integrably bounded (or $\mathbf{L}^p$-bounded) multi-function with values in $\mathbf{R}^m$ and we show that the countable family $\mathcal{F}$ can be chosen to be (relatively) compact in $\mathbf{L}^p$.

Equivalently, we show the existence of a family $\mathcal{F}$ of selections of F describing F as above, such that $\alpha(\mathcal{F}) = 0$, where α is the Kuratowski index. In [**2**], $\alpha(\mathcal{F})$ was determined for the family $\mathcal{F}$ of all the integrable selections of F.

2. Notation and preliminary results. For a subset A of a set X, $(A)^{\mathbf{C}}$ is the complement of A in X. For X a metric space and A bounded, $\alpha(A)$ is the Kuratowski index [**14**],

$$\alpha(A) = \inf\left\{\varepsilon : A = \bigcup_{i=1}^{n} A_i, \mathrm{diam}\,(A_i) \le \varepsilon\right\},$$

$\mathrm{cl}\,(A)$ is the closure of A. The finite dimensional space $\mathbf{R}^m$ is supplied with the norm $\|x\| = \sup_i |x_i|$. $\|A\|$ is $\sup\{\|a\| : a \in A\}$. The closed ball in $\mathbf{R}^m$ of center y and radius ε is denoted by $B[y, \varepsilon]$. When F is a set-valued map and A is a set, $F^{-1}(A)$ is $\{x : F(x) \cap A \ne \varnothing\}$. We recall that F from a measure space to the subsets of $\mathbf{R}^m$ is called measurable if $F^{-1}(A)$ is measurable for every closed A; this implies that $F^{-1}(B)$ is measurable for every open B. For the properties of measurable multi-functions on measure spaces, we refer to [**3**].

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