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GEOMETRIC INEQUALITIES IN NORMED SPACES

S. BUSENBERG AND M. MARTELLI

1. Introduction. In a recent paper [**4**] of S. Busenberg, D. Fisher and M. Martelli, it was shown that the period T of any periodic solution of the first order system

$$\mathbf{x}' = f(\mathbf{x}) \tag{1.1}$$

where $f : \mathbf{E} \to \mathbf{E}$ is Lipschitz continuous with constant L and $\mathbf{E}$ is a normed space, satisfies the inequality

$$TL \geq 6. \tag{1.2}$$

This result refines an earlier estimate of A. Lasota and J. Yorke [**11**], who showed that $TL \geq 4$. Inequality (1.2) is optimal in the stated generality, since Busenberg, Fisher and Martelli [**5**] provided an example where $TL = 6$ in $L^1(Q)$ where Q is the unit square in $\mathbf{R}^2$. In general, the best lower bound for the product TL seems to depend strongly on the geometry of the underlying space. In fact, in the same paper, the three authors gave a simple proof of the better lower bound

$$TL \geq 2\pi \tag{1.3}$$

in spaces with the norm defined via an inner product. Inequality (1.3) was first proved by J. Yorke [**15**]. In [**3**] Busenberg and Martelli give an alternate proof of (1.3) which relies on corresponding inequalities for difference equations in Hilbert spaces. They also show in [**6**] that (1.3) is optimal in every Hilbert space of dimension larger than or equal to two.

Ideas and techniques developed to prove (1.2), (1.3), and to provide the example mentioned above, can be applied to a number of classical problems of geometrical nature to yield different proofs and/or new

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