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ON A MEASURE OF SYMMETRY FOR STATIONARY RANDOM SEQUENCES

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ABSTRACT. Coefficients measuring "departure from exchangeability" are defined and shown to be equivalent to "conditional uniform strong mixing." The result shows that the conditional independence conclusion of the de Finetti theorem has a stability property under "small perturbations" of the exchangeability assumption.

0. Introduction. Part of the celebrated de Finetti theorem says that if $\{X_k\}_{k=1,2,\dots}$ is a random sequence such that $\{X_k\}_{k=1,2,\dots}$ and $\{X_1, \dots, X_n, X_{n+m}, X_{n+m+1}, \dots\}$ have the same distribution for every $n, m \geq 1$, then the random variables $\{X_k\}$ are conditionally independent (see [**1**]). In this note we define "coefficients of symmetry," which measure "departure from exchangeability" of a stationary random sequence. The coefficients, defined by (1) below are nonnegative numbers and equal to zero for exchangeable sequences only. We show that if the coefficients tend to zero, then the distant past and the future of the random sequence become asymptotically "conditionally independent" in the appropriate sense. Namely, we show that a conditional variant of the so-called ϕ-mixing condition and "asymptotic exchangeability" as defined by (1) below are equivalent. Theorem 1 below can be interpreted as stability of the de Finetti theorem.

While there are several other measures of weak dependence used in the literature (for definitions see, e.g., [**5**]), our proof seems to permit only two of them (ϕ-mixing and the so-called ψ-mixing) to be obtained as "measures of departure" from conditional independence in the form given by Theorem 1 below.

Our result might be of some interest to limit theorems, see, e.g., [**6, 8**] for limit theorems under exchangeability; it also might help to

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