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TOPOLOGICAL TYPES OF SEVEN CLASSES OF ISOLATED SINGULARITIES WITH C^*-ACTION

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1. Introduction. In 1982, Mather and the second author [**11**] proved that two germs of complex analytic hypersurfaces of the same dimension with isolated singularities are biholomorphically equivalent if and only if their moduli algebras are isomorphic. It is a natural question to ask for a necessary and sufficient condition for two germs of complex analytic hypersurfaces with isolated singularities (V_1, p_1) and (V_2, p_2) of the same dimension to have the same topological type. We say that (V_1, p_1) and (V_2, p_2) in $\mathbf{C}^{n+1}$ have the same topological type if $(\mathbf{C}^{n+1}, V_1, p_1)$ is homeomorphic to $(\mathbf{C}^{n+1} V_2, p_2)$. Even for $n = 1$, the case is not trivial. It took more than 40 years to get a complete solution. In 1928, Brauner [**2**] proved that the topological type of plane irreducible curve singularity is determined by its Puiseux pairs. In 1932, Burau [**3**] discovered (and also independently by Zariski [**28**]) that for plane irreducible curves the Puiseux exponents are invariant of topological type. Finally, Lejeune [**9**] and Zariski [**26**] proved that the topological type of plane curve singularity is determined by the topological type of all its irreducible components and all the pairs of intersection multiplicity of those components. This together with the theorem of J. Reeve [**17**], which asserts that the intersection multiplicity of two plane curves is the same as the linking number of the corresponding knots, gives a complete answer to our question for $n = 1$.

A polynomial $h(z_0, \dots, z_n)$ is weighted homogeneous of type $(w_0, \dots, w_n)$, where $(w_0, \dots, w_n)$ are fixed positive rational numbers, if it can be expressed as a linear combination of monomials $z_0^{i_0} \dots z_n^{i_n}$ for which $(i_0/w_0) + \cdots + (i_n/w_n) = 1$. $(w_0, \dots w_n)$ is called the weights of h.

Orlik and Wagreich [**15**] and Arnold [**1**] showed that if $h(z_0, z_1, z_2)$ is a weighted homogeneous polynomial in $\mathbf{C}^3$ and $V = \{h(z) = o\}$ has an isolated singularity at origin, then V can be deformed into one of the following seven classes below while keeping the link K_V differentiably

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