ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 22, Number 3, Summer 1992

ON CONTINUED FRACTIONS $K(a_n/1)$, WHERE ALL a_n ARE LYING ON A CARTESIAN OVAL

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ABSTRACT. In a recent paper L. Jacobsen and W.J. Thron have proved results on oval convergence regions and circular limit regions for continued fractions. In the present paper is discussed what happens to the limit region when the oval region is replaced by its boundary. This extends earlier results on boundary versions of Worpitzky's theorem and the parabola theorem.

1. Introduction. The present paper deals with continued fractions

$$\underset{n=1}{\overset{\infty}{\mathbf{K}}} \frac{a_n}{1}, \tag{1.1}$$

and the type of problem to be discussed is: What can be said about the values of (1.1) when all elements a_n are in a prescribed convergence region E? For analytic theory of continued fractions generally, as well as for special concepts like, for instance, "convergence region," we refer to the monograph [**2**]. Following the tradition there, the word "region," used in concepts such as "element region," "convergence region," "value region," "limit region," simply means a *set* of complex numbers. The definition of those concepts are also contained in Section 2 of [**1**].

Our starting point is one of the theorems in [**1**]. We first need to introduce two special types of sets, whose role in continued fraction theory goes back to Lane [**4**].

Let Γ be a complex number with

$$|\Gamma| < |1 + \Gamma| \tag{1.2}$$

Received by the editors on October 10, 1989, and in revised form on December 1, 1989.
1980 *Mathematics Subject Classification.* 30B70, 40A15.
Key words and phrases. Continued fractions, element region, limit region.
This work was supported by the Norwegian Research Council for Science and the Humanities (NAVF).