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SEPARATION THEOREMS FOR NONSELFADJOINT DIFFERENTIAL SYSTEMS

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ABSTRACT. Conditions are given that identify certain solutions of the system of differential equations $x^{(n)} - (-1)^{n-k} \cdot q(t)x = 0$ that must have at least one component that vanishes. Here $q(t)$ is an $m \times m$ matrix of continuous functions that is positive with respect to a certain cone. The results presented are new even for second order self-adjoint systems and for the general scalar equation.

1. Introduction. This paper is concerned with separation theorems for the differential equation

$$x^{(n)} - (-1)^{n-k} q(t)x = 0, \tag{1}$$

where $n \geq 2$ and k is an integer with $1 \leq k \leq n-1$ and where $q(t)$ is an $m \times m$ matrix of functions continuous on the interval $[a,b]$ with $a \geq 0$, subject to the conjugate point type boundary conditions

$$\begin{cases} x^{(i)}(a) = \zeta^i, & i = 0, \dots, k-1, \\ x^{(i)}(b) = \eta^i, & i = 0, \dots, n-k-1. \end{cases} \tag{2}$$

(Also considered is the second order system given by (11) in Section 3, which is more general than (1) for $n = 2$.) More specifically, conditions will be given that identify certain solutions of (1) that must have at least one component that vanishes. Since no assumptions are made on the integer k or on the symmetry of $q(t)$, (1) will in general be nonself-adjoint. But even if (1) is self-adjoint, the results presented here are new. The results are new for the second order case also since the hypothesis on $q(t)$ given in this paper is not as restrictive as that given by the author in [**4**]. The results are also new in the general scalar case.

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